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Accounting for growth: the role of physical work

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Abstract

This paper tests several related hypothesis for explaining US economic growth since 1900. It begins from the belief that consumption of natural resources—especially energy (or, more precisely, exergy) has been, and still is, an important factor of production and driver of economic growth. However the major result of the paper is that it is not 'raw' energy (exergy) as an input, but exergy converted to useful (physical) work that—along with capital and (human) labor—really explains output and drives long-term economic growth. We develop a formal model (Resource-EXergy Service or REXS) based on these ideas. Using this model we demonstrate first that, if raw energy inputs are included with capital and labor in a Cobb–Douglas or any other production function satisfying the Euler (constant returns) condition, the 100-year growth history of the US cannot be explained without introducing an exogenous 'technical progress' multiplier (the Solow residual) to explain most of the growth. However, if we replace raw energy as an input by 'useful work' (the sum total of all types of physical work by animals, prime movers and heat transfer systems) as a factor of production, the historical growth path of the US is reproduced with high accuracy from 1900 until the mid 1 970s, without any residual except during brief periods of economic dislocation, and with fairly high accuracy since then. (There are indications that an additional factor, possibly information technology, needs to be taken into account as a fourth input factor since the 1970s.) Various hypotheses for explaining the latest period are discussed briefly, along with future implications.

JEL classification: 011; 013; 014

Keywords: Exergy; Technology; Economy; Growth; Efficiency; Dematerialisation

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1. Introduction

The primary motivation of this paper is to revisit the neoclassical theory of growth from the physical (thermodynamic) perspective. The 'standard' growth theory, which was formulated in its current production function form (independently) by Solow (1956, 1957) and Swan (1956). The standard theory assumes that production of goods and services (in monetary terms) can be expressed as a function of capital and labor, yet the major contribution to growth had to be attributed to an unexplained exogenous driver called 'technological progress'.

Both casual observation and physical intuition have convinced many investigators since Georgescu-Roegen first expounded on the subject, that production in the real world cannot be understood without taking into account the role of materials and energy (Oeorgescu-Roegen, 1966). Our primary objective in this paper is to elaborate and quantify this intuition—which we share—and to simultaneously endogenize 'technological progress', insofar as possible. A further, though secondary, objective is to clarify the differences between our current approach and the several earlier attempts to incorporate resource flows explicitly into growth models (Jorgenson and Houthakker, 1973; Allen et al., 1976; Hannon and Joyce, 1981; Jorgenson, 1983, 1984). We attempt to explain, hereafter, why the several earlier attempts did not succeed and how—and why—the present approach differs from earlier ones.

Before passing on, we also emphasize that several features of our work follow (albeit indirectly) from our concept of growth dynamics as a positive feedback cycle. This may not seem immediately relevant to our main results. But it is relevant to some of the choices we make later in formalizing the growth model. The generic positive feedback cycle, in economics, operates as follows: cheaper resource inputs, due to discoveries, economies of scale and experience (or learning-by-doing) enable tangible goods and intangible services to be produced and delivered at ever lower cost. This is another way of saying that resource flows are productive, which is our point of departure. Lower cost, m competitive markets, translates into lower prices for all products and services. Thanks to non-zero price elasticity, lower prices encourage higher demand. Since demand for final goods and services necessarily corresponds to the sum of factor payments, most of which go back to labor as wages and salaries, it follows that wages of labor tend to increase as output rises.[2] This, in turn, stimulates the further substitution of natural resources, especially fossil fuels, and mechanical power produced from resource inputs, for human (and animal) labor. This continuing substitution drives further increases in scale, experience, learning and still lower costs.

Based on both qualitative and quantitative evidence, the existence of the positive feedback cycle sketched briefly above implies that physical resource flows must be a major factor of production. Indeed, including a fossil energy flow proxy in the neoclassical production function, without any constraint on factor share, seems to account for economic growth quite accurately, at least for limited time periods, without any exogenous time-dependent term (Hannon and Joyce, 1981; Kümmel, 1982, 1989; Cleveland et al., 1984, 1998; Kümmel

[2] Marx believed (with some justification) that the gains would flow mainly to owners of capital rather than to workers. Political developments have changed the balance of power since Marx's time. However, in either case, returns to energy or physical resources tend to decline as output grows. This can be interpreted as a declining real price.

et al., 1985, 2000; Kaufmann, 1992; Beaudreau, 1998). It is important to note, however, that including energy or exergy as a factor of production does *not* explain economic growth for periods longer than two or three decades, without recalibration or without a time dependent multiplier.[3] The reason for this (negative) empirical result becomes clear hereafter.

The fact that economic growth tends to be very closely correlated with energy consumption, at least for short periods does not a priori mean that energy consumption is the cause of the growth. Indeed, many economic growth models still assume exactly the opposite: that economic growth (due to accumulation of capital, and labor, plus technical progress) is responsible for increasing energy and natural resource consumption. This automatically explains (indeed, guarantees) high correlation. We argue, on the contrary, that declining resource prices can have a direct impact on growth, via the positive feedback loop. The direction of causality must evidently be determined empirically by other means, either theory based or empirical.[4]

The major new feature of our approach is that, in contrast to earlier treatments that introduced (commercial) energy (exergy), or energy (exergy) and materials separately, as factors of production, we consider *physical work* (or '*exergy services*') as the appropriate independent variable for the production function. The term exergy is introduced and explained in Section 2 which follows. It is important to emphasize here that *physical work* is a well-defined concept from thermodynamics and physics; it must be distinguished from the term as it is used in ordinary language, where 'work' is generally what people do to earn a living. The relationship between potential work (exergy) and actual work—or exergy services—performed in the economy is explained in Section 3. In brief, the ratio of actual work to potential work can be interpreted as the *thermodynamic efficiency* with which the economy converts resource inputs into finished materials and services.

To avoid confusion, it is important to note that term 'thermodynamic efficiency', introduced above, is not related to economic efficiency. Thermodynamic efficiency is a straightforward ratio between (physical work) output and resource input. As will be seen, both numerator and denominator are measured in the same physical units (e.g. gigajoules or GJ).

[3] For instance, for the years 1929–1969, one specification that gave good results without an exogenous term for technical progress was the choice of K and E as factors of production. In this case, the best fit ($R^2 = 0.99895$) implied a capital share of only 0.031 and an energy share of 0.976 (which corresponds to very small increasing returns) (Hannon and Joyce, 1981). Another formulation, involving K and electricity, El, yielded very different results, namely ($R^2 = 0.99464$) a capital share of 0.990 with only a tiny share for electricity [ibid]. Using factors K, L only—as Solow did in his pathbreaking (Nobel Prizewinning) paper—but not including an exogenous technical progress factor (as he did) the best fit ($R^2 = 0.99495$) was obtained with a capital share of 0.234 and a labor share of 0.852. These shares add up to more than unity (1.086), which implies significantly increasing returns. Evidently, one cannot rely on econometrics to ascertain the "best" formulation of a Cobb–Douglas (or any other) production function.

[4] There are statistical approaches to addressing the causality issue. For instance, Granger and others have developed statistical tests that can provide some clues as to which is cause and which is effect (Granger, 1969; Sims, 1972). These tests have been applied to the present question (i.e. whether energy consumption is a cause or an effect of economic growth) by Stern (Stern, 1993; Kaufmann, 1995). In brief, the conclusions depend upon whether energy is measured in terms of heat value of all fuels (in which case the direction of causation is ambiguous) or whether the energy aggregate is adjusted to reflect the quality (or, more accurately, the price or productivity) of each fuel in the mix. In the latter case, the econometric evidence seem to confirm the qualitative conclusion that energy (exergy) consumption is a cause of growth. Both results are consistent with the notion of mutual causation.

Moreover, we are able to estimate both inputs and outputs, and the resulting ratio, with reasonable accuracy, from published empirical data (see Sections 2 and 3).

Introducing an additional factor creates certain conceptual problems that we must acknowledge from the outset. Suppose we had opted (as some modelers have) to choose exergy inputs as a factor of production, measured in monetary terms. Payments for fossil fuels, minerals, ores, farm products and other forms of ‘raw’ exergy inputs are actually payments for ‘produced’ outputs of the extractive industries, agriculture and forest products sectors. By convention, all of these are intermediates, accounting for a very small percentage of GDP—perhaps 4% without agriculture and less than 10% even if agriculture is included. Evidently, electric power, motive power, space heat and industrial heat are also produced outputs. Of course, some capital and labor are required to produce these intermediate products.

However, among these exergy services only electric power is regarded as a *commodity* produced and sold by a well-defined industrial sector for which financial accounts are kept. Motive power is produced and consumed (mostly) within the agriculture, transportation and construction sectors, while heat is produced and consumed within many other sectors, including households. They are not regarded as (or, accounted for) commodities, and they do not have explicit market prices. If shadow prices for these kinds of exergy services (useful work) were available, it is likely that the corresponding payments would account—*in toto*—for a considerably greater share of the US GDP. But, needless to say, capital and labor, as well as inputs from the extractive and farming sectors, are also required to produce these intermediates, just as they, in turn, are required to produce other goods and services.

In short, to introduce either ‘raw’ exergy or exergy services as a third factor of production also forces us to think in terms of a multi-sector input-output structure with inter-industry feedbacks. The two choices (exergy or exergy services) differ only in the magnitudes of the feedbacks from downstream products and services back to extraction and primary processing. At first glance this might argue against introducing either of them as a third factor.

Note that capital goods are also produced intermediates. The inputs to capital goods production are—again—capital, labor and other intermediates (including exergy and/or exergy services). The key conceptual difference is that capital goods and labor are not consumed in the production process[5] (although depreciation is almost a form of consumption), whence they are *cumulable*, and capital and labor services are proportional to the corresponding *stocks*. On the other hand, resource (exergy) flows, or exergy service flows, are not cumulable; they are consumed immediately in the production process.

Furthermore, thanks to cumulability, capital services and labor services can be—within limits—regarded as *independent* variables in the sense of being independent of current economic conditions (i.e. demand *vis a vis* potential supply). Of course, the true relationship between capital and output is one of mutual dependence, but with a time lag between the output level and the stock levels. It takes a few years for capital stocks to respond to current economic conditions via the price mechanism. The potential labor supply responds through demographic feedbacks over an even longer time frame, whence adjustment of current labor supply occurs mainly through the political process (i.e. laws regarding minimum schooling

[5] Georgescu-Roegen was the first to have emphasized this crucial point (Georgescu-Roegen, 1971).

requirements, retirement ages, work-weeks, immigration, and so on). On the other hand, both resource (exergy) consumption, and exergy service (useful work) consumption levels respond rather quickly to economic conditions (via prices), whereas the forward impact of changes in prices on demand—up or down—driven by technological improvements and/or resource scarcity lags by several years.

Having acknowledged these points, the question arises: do they, taken together, preclude the use of exergy flows or exergy service flows as inputs to a formal production function? We think that the answer is 'no'. We argue (Section 4) that the economic system should be understood as a sequential materials processing system, converting raw materials (and fuels) by stages into final products and services. The existence of possibly lagged) feedbacks from downstream sectors to upstream sectors is understood. Capital services constitute one such lagged feedback. Exergy services can be regarded as a generic intermediate with both feedback and feed-forward. Whether it has explanatory power is then an empirical question.

Section 5 presents the formal Resource-EXergy Service (REXS) model, which is mainly defined by a choice of variables and production function. Section 6 presents the main results and Section 7 summarizes and discusses further implications.

2. The role of natural resources and energy (exergy)

An obvious implication of economic history—and one that is consistent with our view of growth dynamics as a feedback cycle—it that important 'engine of growth' since the first industrial revolution has been the continuously declining real price of physical resources, especially energy (and power) delivered at a point of use. The tendency of virtually all raw material and fuel costs to decline over time (lumber was the main exception) has been thoroughly documented, especially by economists at Resources For the Future (RFF) (Barnett and Morse, 1962; Potter and Christy, 1968; Smith and Knitilla, 1979). The increasing availability of energy from fossil fuels, and power from steam engines and internal combustion engines (ICEs), has clearly played a fundamental role in past economic growth. Machines powered by fossil energy have gradually displaced animals, wind power, water power and human muscles and thus made human workers vastly more productive than they would otherwise have been. There is no dispute among economists on this point.

The term *energy* as used above, and in most discussions (including the economics literature) is actually technically incorrect. The reason is that *energy* is conserved in every activity or process and therefore cannot be 'used up'—as most common usages of the term imply. But energy is not necessarily available to do useful work. The standard textbook example is the heat energy in the ocean water, virtually none of which can be utilized fordoing useful work. As was discovered nearly two centuries ago by the French engineer Sadi Carnot, heat can only be converted into useful work if there is a temperature *gradient*. Absolute temperature does not matter. It is the temperature difference between two reservoirs that determines the amount of work that can be extracted by a so-called heat engine. By the same token, it is the temperature difference between the sun and the earth that drives most natural processes on earth, including the weather and photosynthesis.

Exergy is the correct thermodynamic term for 'available energy' or 'useful energy', or energy capable of performing mechanical, chemical or thermal work. The distinction is theoretically important because energy is a conserved quantity: this is the famous first law of thermodynamics. Energy is not ‘used up’ in physical processes, it is merely degraded from available to less and less available forms. On the other hand, exergy is dissipated (used and destroyed) in all transformation processes. The measure of exergy destruction is the production of a thermodynamic quantity called *entropy* (second law of thermodynamics).

The formal definition of exergy is the maximum work that could theoretically be done by a system as it approaches thermodynamic equilibrium with its surroundings, reversibly. Thus, exergy is effectively equivalent to *potential work*. There is an important distinction between *potential work* and *actual work done* by animals or machines. The *conversion efficiency* between exergy potential work), as an input, and actual work done, as an output, is also an important concept in thermodynamics. The notion of thermodynamic efficiency plays a key role in this paper.

To summarize the technical definition of exergy is the maximum work that a subsystem can do as it approaches thermodynamic equilibrium (reversibly) with its surroundings. Exergy is also measured in energy units, and exergy values are very nearly the same as *enthalpy* (heating values) for all ordinary fuels. So, effectively, it is what most people mean when they speak of ‘energy’, the major exception to this rule is that exergy is a measure that is applicable, and can be estimated with acceptable accuracy, not only for traditional fuels but to all agricultural products and industrial materials, including minerals. This point is important because it enables us to construct an aggregate measure of all resource flows into the economic system, as well as an aggregate measure of all processed intermediate flows. We have tabulated and published exergy values per kilogram for most common materials and mixtures (such as ores) in (Ayres and Ayres, 1999). See Appendix A of this paper for more details.

3. Physical work and thermodynamic conversion efficiency

As noted above, exergy is equivalent to maximum *potential work*. There are several kinds of work, including mechanical work, electrical work and chemical work. For non-engineers, mechanical work can be exemplified in a variety of ways, such as lifting a weight against gravity or compressing a fluid. The term horsepower was introduced in the context of horses pumping water from flooded 18th century British coal and tin mines. A more general definition of work is movement against a potential gradient (or resistance) of some sort. A heat engine is a mechanical device to perform work from heat (though not all work is performed by heat engines).

With this in mind, we can subdivide work into three broad categories, as follows: work done by animal (or human) muscles, work done by heat engines or water or wind turbines and work done in other ways (e.g. thermal or chemical work). Mechanical work can be further subdivided into work done to generate electric power and work done to provide motive power (e.g. to drive motor vehicles). The power sources in this case are so-called ‘*prime movers*’, including all kinds of internal and external combustion engines, from steam turbines to jet engines. So called ‘renewables, including hydraulic, nuclear, wind and solar power sources for electric power generation are conventionally included. However electric motors are *not* prime movers, because electricity is generated by some other prime mover, usually a steam or gas turbine. In fact, electricity can be defined (for purposes of this paper) as ‘pure’ work.

Chemical work is exemplified by the reduction of metal ores to obtain the pure metal, or indeed to drive any endothermic chemical synthesis process (ammonia synthesis is a good example). Thermal work is exemplified by the transfer of heat from its point of origin (e.g. a furnace) to its point of use, via one or more heat exchangers and a carrier (such as steam, hot water or hot air).

To measure the useful work *U* done by the economy, in practice, it is helpful to classify fuels by use. The first category is muscle work, for which the fuel is food or feed. In the US, human muscle work was quantitatively insignificant by 1900 and can be neglected. Horses and mules, which accounted for most animal work on US farms and urban transport, have not changed significantly since then. Animal work was still significant up to the 1930s but mechanical and electrical work have since become far more important. The thermodynamic efficiency with which horses and mules convert feed energy to useful work is generally reckoned at about 4% (i.e. one unit of work requires 25 units of feed).

The second category is fuel used by prime movers to do mechanical work. This consists of fuel used by electric power generation equipment and fuel used by mobile power sources such as motor vehicles, aircraft and so on. As regards mobile power sources, we define thermodynamic efficiency in terms of useful work performed by the whole vehicle, against air resistance and rolling resistance of the wheels on the road, not just work done by the engine itself. Thus, the efficiency of an automobile is the ratio of work done by the vehicle to the total potential work (exergy content) of the fuel.

The third broad category is fuel used to generate heat *as such*, either for industry process heat to do chemical work) or space heat and domestic uses such as washing and cooking. The efficiency, in this case, refers to the delivery system. Lighting can be thought of as a special case of heating. Clearly, the efficiency of muscles as energy converters has not changed during human history. But the conversion efficiency of heat engines, domestic and commercial heating systems and industrial thermal processes has increased significantly over the past 100 years. We have plotted these increasing conversion efficiencies, from 1900 to 1998 in Fig. 1. Detailed derivations of these curves involve extensive reviews of the engineering literature and technological history. Details, including data sources, can be found in another publication (Ayres and Warr, 2003).

Electrical work output is measured directly in kilowatt-hours (kWh) generated. Data are published by the US Federal Power Commission and the US Department of Energy (see Appendix A). Other types of work must be estimated from fuel inputs, multiplied by conversion efficiencies, as shown in Fig. 1, over time. Allocations of fossil fuel exergy inputs to the economy by type of work are shown in Fig. 2. Electrification has been perhaps the single most important source of useful work for production of goods and services, and (as will be seen later) the most important single driver of economic growth during the twentieth century. The fuel exergy required to generate a kilowatt-hour of electric power has decreased by a factor of ten during the past century. This implies that the thermodynamic efficiency of conversion increased over that period by the same factor, as shown in Fig. 2.

Electricity prices fell correspondingly, especially during the first half of the century. However, the consumption of electricity in the US has increased over the same period by

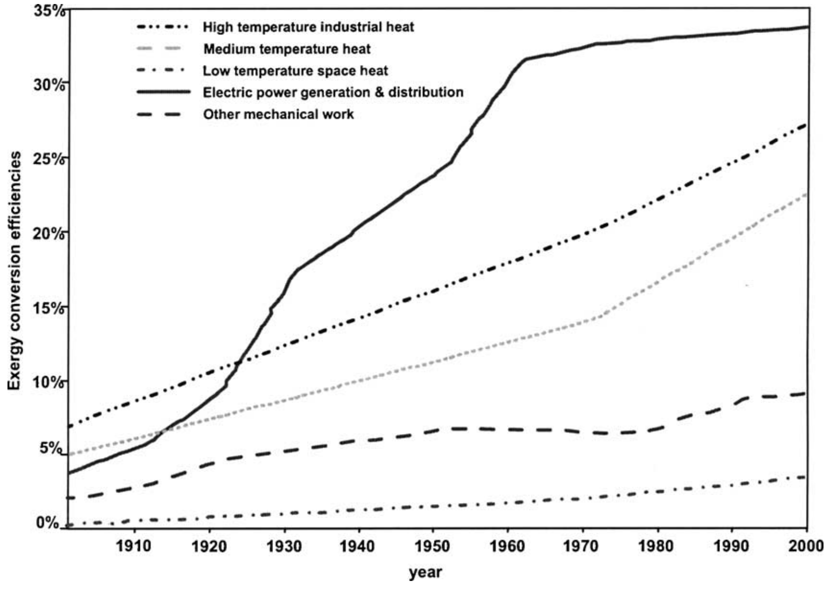

Fig. 1. Energy (exergy) conversion efficiencies, USA, 1900–1998.

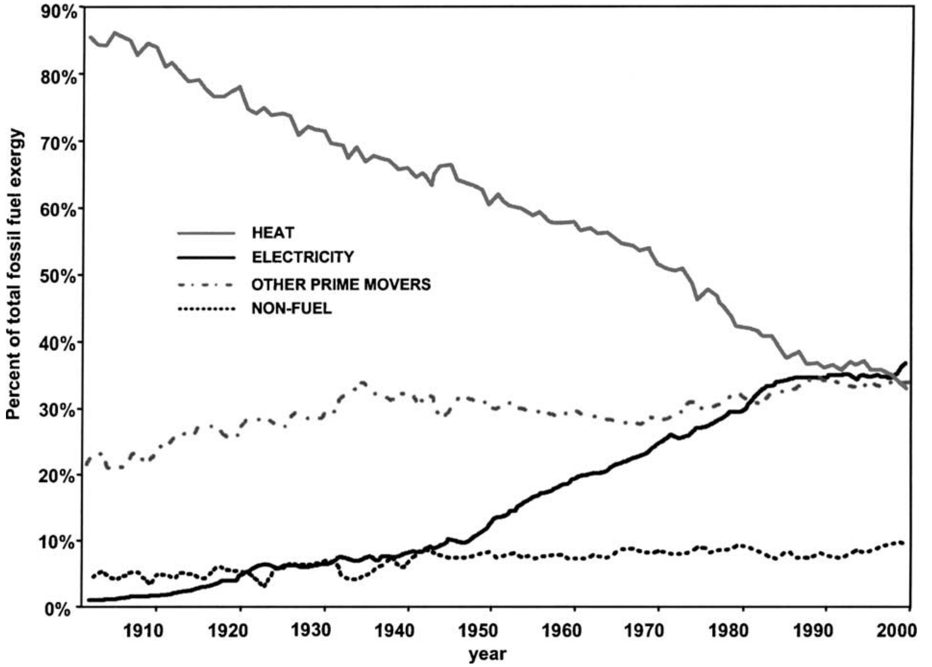

Fig. 2. Fossil fuel consumption exergy allocation, USA, 1900–1998.

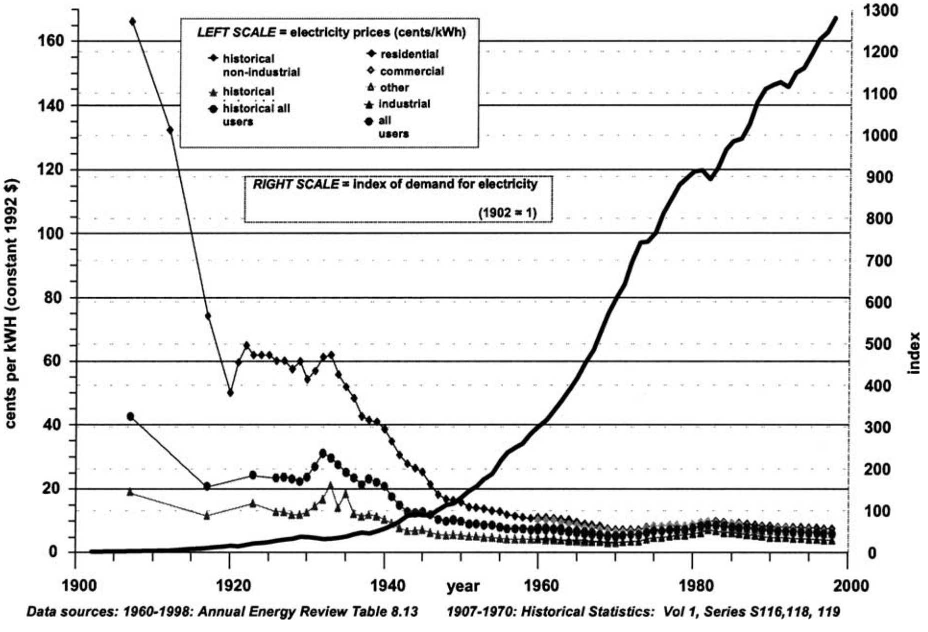

Fig. 3. Electricity prices and electrical demand, USA, 1900–1998.

a factor of more than 1300, as shown in Fig. 3. (This exemplifies the positive feedback economic 'growth engine' discussed briefly in the introduction.)

4. Towards a new theory of production and growth

Before proceeding further, it is important to mention one of the key assumptions of the standard theory, as set forth by Robert Solow, namely that marginal factor productivity can safely be equated with factor share in the national accounts. This simplistic assumption is particularly convenient for models based on Cobb–Douglas production functions. It is built into virtually all textbook discussions of growth theory, since the implications for labor and capital (marginal) productivity are easily derived. Labor gets the lion's share of payments in the US national accounts, around 70%. Capital (defined as interest, dividends, rents and royalties) gets all of the rest, because all payments are attributable to one category or the other, by definition. The figures vary slightly from year to year, but they have been relatively stable for the past century or more. It follows that marginal labor productivity should be around 0.7 and marginal capital productivity should be around 0.3 in a Cobb–Douglas framework.

Payments to extractive resource owners (excluding farms) are hidden in the capital accounts, and they constitute a very small proportion—perhaps 34% of GDP. This implies that resource productivity must be correspondingly small in comparison with labor or capital productivity. This has been a major source of confusion and misdirected effort in the past.

We reject this simple assumption (along with most modern modelers) on the basis of two arguments. The first follows from our view of the growth process as a positive feedback cycle, as discussed previously. This implies that resource (exergy) flows—or, more precisely, declining resource prices—are *not* simply a consequence of growth. They are also (and simultaneously) a cause of growth. This means that the marginal productivity of resource flows should not be quantitatively insignificant compared to the marginal productivities of other factors. Nor should it be constant over a long period of time. There is an apparent inconsistency between very small factor payments directly attributable to physical resources—especially fossil fuels—and the obvious importance of energy (exergy) as a factor of production.

The second argument, which is more rigorous, is based on the fact that the identification of marginal factor productivities with factor shares in the national accounts is based on an oversimplification of the neoclassical theory of optimal income allocation. If labor and capital are the only two factors of production, neoclassical theory implies that the productivity of a factor of production must be proportional to the share of that factor in the national income. This proposition is quite easy to prove in a hypothetical single sector economy consisting of a large number of producers manufacturing a single all-purpose good using only labor and capital services. The textbook example is usually bread, produced by bakeries that produce bread from capital and labor, but without any inputs of flour or fuel (Mankiw, 1997).

The supposed link between factor payments and factor productivities gives the national accounts a direct and fundamental (but spurious) role in production theory. In reality, however (as noted in the introduction), the economy produces final products from a chain of intermediates, not directly from raw materials or, still less, from labor and capital without material inputs. In the simple single sector model used to 'prove' the relationship between factor productivity and factor payments, this crucial fact is neglected. Allowing for the omission of intermediates (by introducing even a two- or three-sector production process) the picture changes completely. In effect, downstream value-added stages act as productivity multipliers. This enables a factor receiving a very small share of the national income directly, to contribute a much larger effective share of the value of aggregate production, i.e. *to be much more productive* than its share of overall labor and capital would seem to imply if the simple theory of income allocation were applicable (Ayres, 2001).

Our rejection of the simplistic identification of marginal productivities with factor shares has two consequences. One is that we are free to depart from the Cobb–Douglas strait-jacket. The other is that we must determine the parameters of the chosen production function by means of statistical fitting procedures. These issues are discussed in the next section.

For clarity in further discussion, we use the conventional terminology L for human labor, as indexed to man-hours employed, and K for produced capital (a construct of accumulated investment less depreciation), as compiled and published by the Bureau of Economic Analysis in the US Department of Commerce. We use the symbol E for the energy inputs to the economy, as traditionally defined and compiled by the US Department of Energy. This consists of the heat (actually, *enthalpy*) content of fossil fuels and fuelwood, plus the nuclear heat used as an input to nuclear electric power generation, and the energy of flowing water harnessed for purposes of hydro-electric power production, plus small contributions from wind and solar heat. This variable has been used many times in the economics literature.

We use the symbol *B* for exergy inputs to the economy, which include the items above—all of which are *potential* (but not actually performed) work—plus the potential work embodied in non-fuel wood and agricultural products and non-fuel minerals, such as sulfide ores. In practice, the mineral contribution to exergy is quite small (except in the metallurgical industry itself) and can be neglected without significant error. The major quantitative difference between *E* and *B* nowadays is that the latter is slightly larger and more inclusive. However, in the 19th century and the early years of the 20th century (and in many developing countries) the differences are significant.

We use the symbol *U* for work actually performed in the economy for economic purposes. The components of performed work include animal work (by horses and mules), work done by prime movers (both electric power generated and motive power) and heat delivered to a point of use, whether industrial or residential. The human contribution to physical (muscle) work can be neglected in comparison to other inputs as a first approximation.[6] We distinguish two variants of performed work, namely U_E and U_B where the second variant includes animal work, whereas the first variant does not. The distinction is necessary because animals consume feed produced by the agriculture sector, which is included in *B* but not included in the conventional measure of energy inputs tot he economy, *E*.

Given the assumed importance of resource (exergy) flows in the economy, one might postulate two simple linear relationships of the form:

$$Y = f_E g_g E = g_E U_E \tag{1a}$$

$$Y = f_B g_B B = g_E U_E \tag{1b}$$

where *Y* is GDP, measured in dollars, *E* is a measure of commercial energy (mainly fossil fuels), *B* is a measure of all 'raw' physical resource inputs (technically, exergy), including fuels, minerals and agricultural and forest products. Then *f* is the thermodynamic efficiency defined earlier, namely the ratio of 'useful work performed' *U* done by the economy as a whole to 'raw' exergy input. Then *g* is the ratio of economic output in value terms to useful work performed. The variables *f*, *g* and *U* have implicit subscripts *B* or *E*, which we neglect hereafter where the choice is obvious from context. Since work appears in both numerator and denominator, its definition depends on whether we choose *B* or *E*. Note that Eqs. (1a) and (1b) are essentially definitions of the two new variables *f* and *g*. There is no theory or approximation involved at this stage, except for the implicit assumption of linearity.

[6] The US population in 1900 was 76 million, of which perhaps 50 million were of working age. but only 25 million were men (women worked, without question, but their work did not contribute to GDP at the time), and at least half of the male workers were doing things other than chopping wood, shoveling coal or lifting bales of cotton, which depended more on eye-hand coordination or intelligence than muscles. The minimum metabolic requirement is of the order of 1500 cal per day (for men), whereas the average food consumption for a working man was about 3000 cal per day, whence no more than 1500 cal per day was available for physical work. This comes to 18 billion cal per day or about 0.16 EJ per year of food exergy inputs for work, compared to fossil fuel consumption of 8.9 EJ in that year. If muscles convert energy into work at about 15% efficiency, the overall food-to-work conversion efficiency for the human population as a whole would also be roughly 2.4%. In recent years, though most women have jobs, given the changing nature of work, and the much greater life expectancy and retirement time, the conversion efficiency has declined significantly.

There are two mathematical conditions that a production function must satisfy to be economically realistic. One is the condition of constant (or nearly constant) returns to scale. This implies that the function should be a first-order homogeneous function of its variables (known as the Euler condition). The other requirement is that the logarithmic derivatives (marginal productivities) of the factor variables should be non-negative—at least on average—throughout the entire time period (1900–1998).

Subject to these requirements, we note that the expressions (1a) or (1b) can be converted into a production function in either of two ways. The first possibility is to specify either E or B as a plausible factor of production (along with K and L). Then the product fg with subscripts E or B can be approximated by some first order homogeneous function of the three factors: labor L, capital K and E or B. The second possibility is that useful work U is a more plausible factor of production (instead of E or B) and the function g can be expressed approximately by some first order homogeneous function of K, L and U, again with appropriate subscripts We have tested these possibilities empirically, for several choices of production function, as noted hereafter.

To begin with, the traditional variables capital K, and labor L, as usually defined for purposes of economic analysis, are plotted in Fig. 4 from 1900 to 1998, along with deflated GDP and a traditional Cobb–Douglas production function of K, L and E. It is important to note that GDP increases faster than any of the three contributory input factors. The

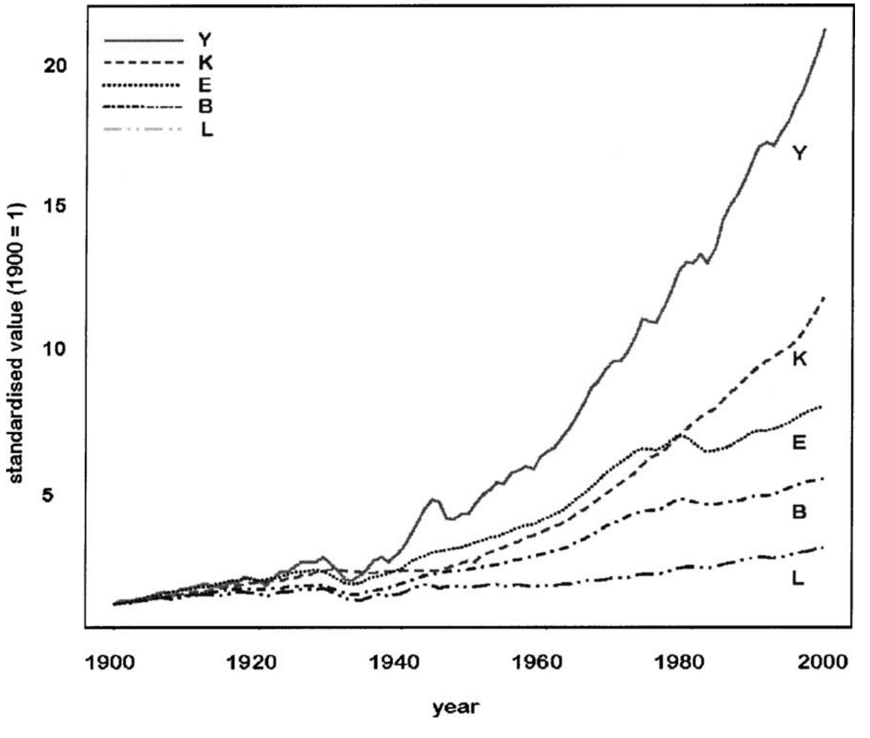

Fig. 4. GDP (Y) and factors of production K, L, B and E: USA, 1900–1998.

need for a time-dependent factor representing technical progress (i.e. the Solow residual) is evident as seen in the figure. Replacing *E* by *B* (i.e. including biomass) does not affect that qualitative conclusion. Substituting a more complex form of production function, whether CES, trans-log or linear-exponential (LINEX) (introduced later) with the same variables does not make a significant difference in the need for an exogenous multiplier, although the unexplained residual can be reduced slightly. The problem is, simply, that US GDP since 1900 has increased faster than *K* and *L* or either *E* or *B*, and therefore *faster than any homogeneous first order combination of those variables*. Thus, from here on, we drop the possibility of using either *E* or *B* as a factor of production in a production function.

We now turn to the alternative possibility, namely to try useful work *U* (exergy services) as a factor of production instead of *E* or *B*. The analogy with capital services seems apposite. Effectively there are two definitions of useful work to be considered hereafter, namely

$$U_B = f_B B \tag{2a}$$

or

$$U_E = f_E E \tag{2b}$$

As mentioned earlier, the ratios *f* are, effectively, composite overall thermodynamic exergy conversion efficiencies. The former takes into account animal work and agricultural products, including animal feed. The latter neglects animal work and agricultural production. These two aggregate efficiency trends are calculated using exergy input data from 1900 to 1998, as shown in Fig. 5 multiplied by estimated thermodynamic conversion

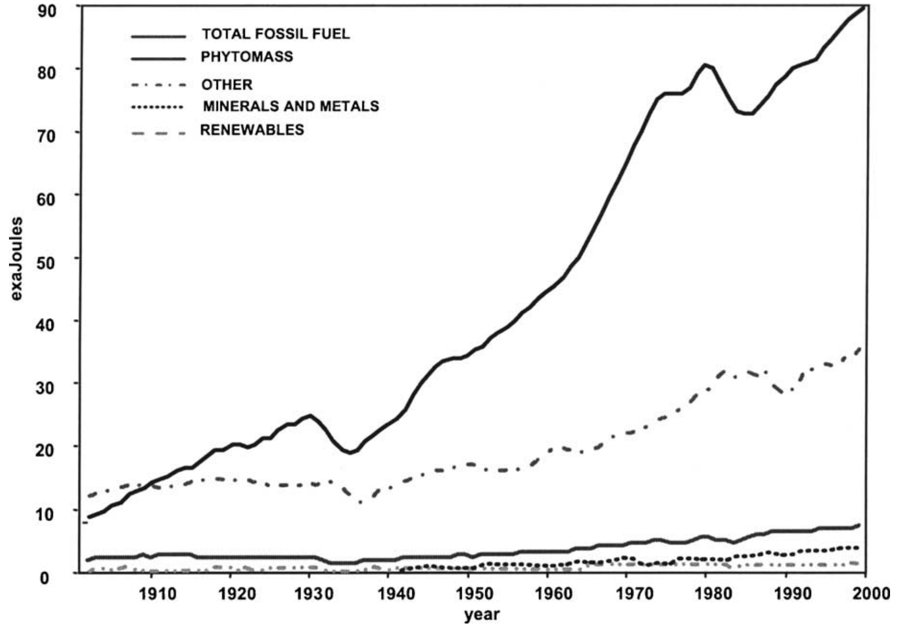

Fig. 5. Exergy inputs to the US economy, 1900–1998.

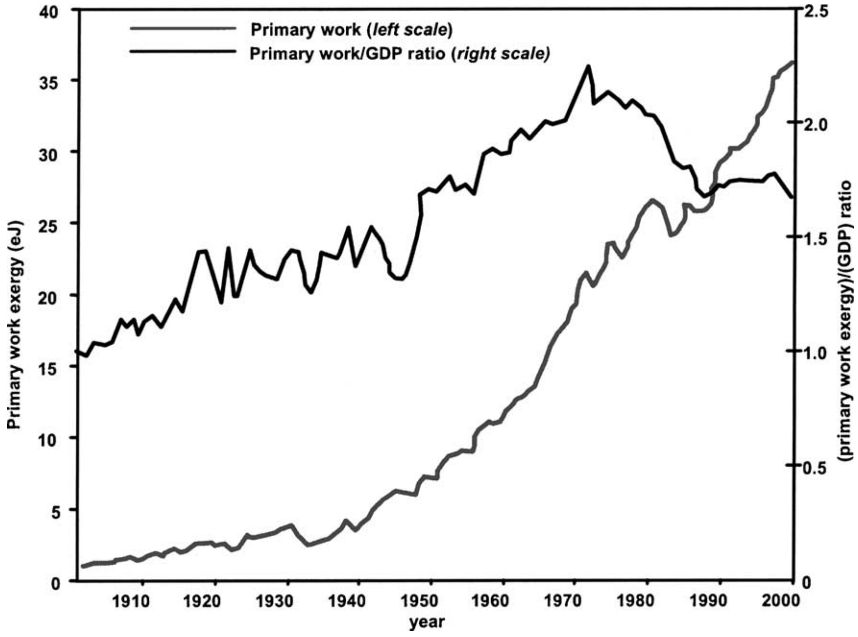

Fig. 6. Primary work and primary work/GDP ratio, USA, 1900–1998.

efficiency trends plotted by type of useful work, in Fig. 1. Evidently, if the trends in f_E and f_B are fairly steadily upward throughout a long period (such as a century) it would seem reasonably safe to project these trend curves into the future for two or three decades. The trends in physical work performed (U_B) and work/GDP ratio are shown in Fig. 6.

5. Choosing a production function

Having decided to introduce the broadest definition of useful work (or exergy services) U_B as a factor of production, in addition to the usual capital K and labor L, the choice of functional forms remains. As noted already, the Cobb–Douglas form is attractive in the case of two factors K and L because the marginal productivities of capital and labor of can be equated with factor shares in the national accounts. But adding a third factor that is not independent of the other two, invalidates this argument. At first glance, this is unfortunate, because it means the parameters of the production function—whether Cobb–Douglas, CES, trans-log or other—will have to be determined by statistical fitting methods, with all the associated difficulties.

On the other hand, giving up the restrictive Cobb–Douglas assumption of constant marginal productivities over long time periods has a compensating advantage: it means that a better fit may be possible than pure Cobb–Douglas would allow. (In fact, this hope is realized). Actually, the form of production function we have used was originally derived by reversing the usual logic (Kümmel et al., 1985): Instead of choosing a mathematical production function and performing logarithmic differentiation, one can choose simple mathematical forms for the three marginal productivities (based on plausible assumptions about asymptotic behavior), and perform three partial integrations instead. The resulting LINEX form is given below. We have merely substituted U_B for E in Kümmel's function, yielding

$$Y = AU \exp\left(\frac{aL}{U} - \frac{b(U+L)}{K}\right) \tag{3}$$

where a and b are parameters to be chosen econometrically and A is a multiplier. If economic output and growth are fully explained by the three variables, then the multiplier A should be independent of time. It can be verified without difficulty that the R.H.S. of (3) satisfies the Euler condition for constant returns-to-scale. It can also be shown that the requirement of non-negative marginal productivities can be met.

As a matter of fact, the LINEX function has another useful feature that is worthy of mention. Namely, it does not imply (as does the Cobb–Douglas function) that the three factors are all strict substitutes for each other in the sense that more of one factor implies less of the other, or conversely. On the contrary, it implies a more complex and more realistic substitution–complement relationship among the variables.

The three factor productivities are easily derived by differentiation as follows:

$$\frac{\partial Y}{\partial K}\frac{K}{Y} = \frac{bL}{K} \tag{4a}$$

$$\frac{\partial Y}{\partial L}\frac{L}{Y} = \frac{aL}{U} - \frac{bL}{K} \tag{4b}$$

$$\frac{\partial Y}{\partial U}\frac{U}{Y} = 1 - \frac{aL}{U} - \frac{bU}{K} \tag{4c}$$

The requirement of non-negativity is equivalent to the following three inequalities:

$$b > 0 \tag{5a}$$

$$aK > bU \tag{5b}$$

$$1 > \frac{aL}{U} + \frac{bU}{K} \tag{5c}$$

The first condition (5a) is trivial. However, the second and third conditions are not automatically satisfied for all possible values of the variables. It is therefore necessary to do the fitting by constrained non-linear optimization. The statistical procedures and quality measures are discussed in Appendix B.[7]

[7] In recent work subsequent to the submission of this paper, we have carried out a large number of statistical tests for both Cobb–Douglas and LINEX functions. The standard procedure is to carry out the OLS fit for annual increments, to eliminate possible collinearity. The results are essentially the same as those presented here.

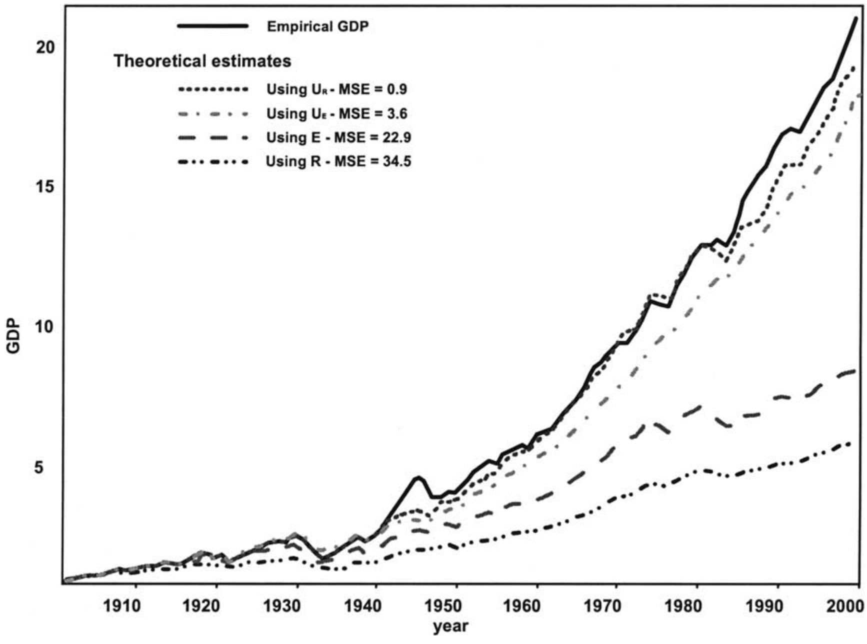

Fig. 7. LINEX production function fits with different “energy” factor inputs, USA, 1900–1998.

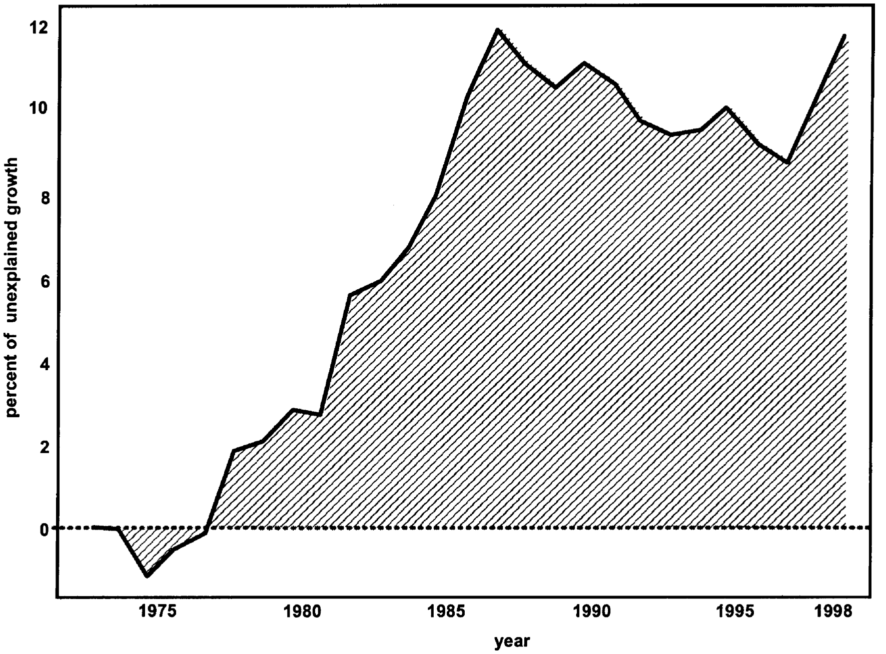

Fig. 8. Percentage of growth unexplained by LINEX fit with U_B, USA, 1900–1998.

6. Results

The two curves in Fig. 7 show the LINEX fit, with work U_E and U_B respectively, as factors of production. In the first case, we consider physical work from commercial energy sources U_E (excluding animal work) as a factor. The second case U_B, reflects work derived from all exergy inputs (B includes animal work derived from agricultural phytomass). The best fit, by far, is the latter. The unexplained residual has essentially disappeared, prior to 1975 and remains small thereafter. In short, it would seem that 'technical progress'—as defined by the Solow residual—is almost entirely explained by historical improvements in exergy conversion (to physical work), as summarized in Fig. 2, at least until recent times. The remaining unexplained residual, roughly 12% of recent economic growth (since 1975), is shown next in Fig. 8.

We conjecture that a kind of phase-change or structural shift took place at that time, triggered perhaps by the so-called energy crisis, precipitated by the OPEC blockade. Higher energy prices induced significant investments in energy conservation and systems optimization. For instance, the CAFE standards for automobile fuel economy, introduced in the late 1970s, forced US motor vehicle manufacturers to redesign their vehicles. The result was to double the vehicle miles obtained from a unit of motor fuel in the US between 1970 and 1989. This was achieved mainly by weight reduction and improvements in aerodynamics and tires. Comparable improvements have been achieved in air travel, rail freight and in many manufacturing sectors, induced primarily by the sharp (though temporary) fuel price increases.

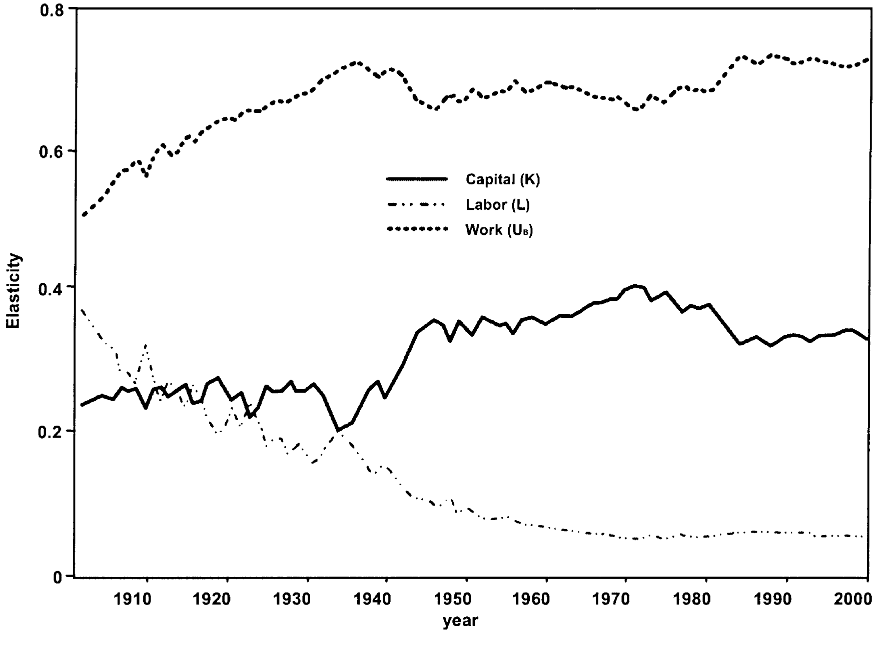

Fig. 9. Marginal productivities (elasticities) of each factor of production using U_B, USA, 1900–1998.

The marginal productivities of the factors can be calculated directly from Eq. (4).

The three marginal productivities for the preferred case, U_B are plotted in Fig. 9. The marginal productivity trends for capital and work, in both cases show a very slight directional change between 1970 and 1980. The marginal productivity of capital has started to increase whereas the marginal productivity of physical work—resulting from increases in the efficiency of energy conversion—has declined slightly. This shift roughly coincides with the two so-called oil crisis, and may well have been triggered by the spike in energy (exergy) prices that occurred at that time.

7. Summary and conclusions

In the 'standard' model a forecast of GDP requires a forecast of labor L, capital stock K and the Solow multiplier—multifactor productivity or technical progress—$A(t)$. We have shown that introducing energy and/or material resource (i.e. exergy) inputs does not significantly improve the explanatory power of traditional production functions. A time-dependent Solow-multiplier is still needed.

However, a much better explanation of past economic growth can be obtained by introducing exergy services (useful work) as a factor of production, in place of exergy inputs.

Exergy services can be equated to exergy inputs multiplied by an overall conversion efficiency which, of course, corresponds to cumulative technological improvements over time. Based on this hypothesis economic growth from 1900 to 1975 or so is explained almost perfectly, except for wartime perturbations. The results described above, the technical progress term can be decomposed into two main contributions. The most important, historically, is from improved exergy conversion-to-work efficiency. This propagates, via cost and price reductions, through the whole downstream value-added chain.

More recently, however, there is obviously some contribution from 'other' downstream technical improvements. Evidently growth of GDP in the past quarter century has slightly outstripped growth of the three main input factors, capital, labor and physical work. Since 1975 or so an additional source of value-added is involved. One possibility is energy conservation and systems optimization triggered by the energy (exergy) price spike in the 1973–1981 period. The other obvious candidate for this additional value creation is information and communications technology (ICT). However, in the spirit of some endogenous growth theories, it would be possible to interpret this additional productivity to some qualitative improvement in either capital or labor.

It does appear that the marginal productivity of physical work is still by far the dominant driver of past growth and will be for decades to come. This does not mean that human labor or capital are unimportant. As noted already, the three factors are not really independent of each other. Increasing exergy conversion efficiency requires investments of capital and labor, while the creation of capital is highly dependent on the productivity of physical work.

It is tempting to argue that the observed shift starting in the 1970s reflects the influence of information technology. Certainly, large scale systems optimization depends very strongly on large data bases and information processing capability. The airline reservation systems now in use have achieved significant operational economies and productivity gains for airlines by increasing capacity utilization. Manufacturing firms have achieved comparable gains in machine utilization and inventory control through computerized integration of different functions.

One of the more important implications of the foregoing is that some of the most dramatic and visible technological changes of the past century have *not* contributed significantly to overall economic growth. An example in point is medical progress. While infant mortality has declined dramatically and life expectancy has increased very significantly since 1900, it its hard to see any direct impact on economic growth, at least up to the 1970s. Neither of the two major benefits adds to labor productivity. The gain has been primarily in quality of life, not quantity of output. Although health services demand an increasing share of GDP, there is no indication of a decline in prices, as implied by the positive feedback ‘growth engine’.

Changes in telecommunications technology since 1900 may constitute another example. New service industries, like moving pictures, radio and TV have been created, but if the net result is new forms of entertainment, the gains in employment and output may have come largely at the expense of earlier forms of public news and entertainment, such as the print media, live theater, circuses and vaudeville. Again, the net impact may have been primarily on quality of life. While the changes have been spectacular, as measured in terms of information transmitted, the productivity gains may not have been especially large, at least until recently (the 1990s) when the internet began to have an impact on ways of doing business.

In any case, since economic growth for the past century can be explained with considerable accuracy by three factors K, L and U_B, it is not unreasonable to expect that future growth for some time to come will be explained quite well by these same variables, plus a small but growing contribution from ICT.

From a long-term sustainability viewpoint, this conclusion carries a powerful implication. If economic growth is to continue without proportional increases in fossil fuel consumption, it is vitally important to exploit new ways of generating value added without doing more work. But it is also essential to develop ways of reducing fossil fuel exergy inputs per unit of physical work output (i.e. increasing conversion efficiency). In other words, energy (exergy) conservation is probably the main key to long term environmental sustainability.

Appendix A. Data

We have compiled a number of historical data sets for the US from 1900 through 1995, indexed to 1900. All of the series are from standard sources. Both labor and capital series up to 1970 are found in (USDOCBEA, 1973) *Long Term Economic Growth 1860–1970*, US Department of Commerce, Bureau of Economic Analysis. Tables (Series A-68 and A-65, respectively). More recent data (1947–1995) came from (USCEA, 1996) *Economic Report of the President, 1996* (Tables B-32 and B-43). The earlier and later labor series are not exactly the same, but the differences during the period of overlap (1949–1970) are very minor. The capital series since 1929 comes from (USDOCBEA, monthly) *Survey of Current Business*, May 1997, and (USDOC, 1992) *Business Statistics*, also the US Department of Commerce. Labor is counted as man-hours actually worked, and private reproducible capital stock, adjusted by the fraction of the labor force actually employed. This same adjustment was also made by Solow (1957).

The exergy series are much more complicated. In brief, we have compiled historical data on fuel consumption for all fuels, including wood, and for non-fuel material inputs with non-trivial exergy content, including non-fuel wood, and major metal ores (iron, copper) and minerals (limestone). Data for 1900 to 1970 are mostly from (USCensus, 1975) *Historical Statistics of the US from Colonial Times to 1970*, various tables, with some interpolations and estimates for missing numbers. More recent data on fuels—both raw and processed (including electricity)—are from (USDOEEIA, annual) US Department of Energy, *Annual Review of Energy Statistics*. Data on other minerals and metal ores are from (USGS, 1999; USBuMines, annual) *Minerals Yearbooks* (US Bureau of Mines until 1995; US Geological Survey since then). We have calculated the exergy for all fuels as a multiplier of heat content; exergy for other materials was calculated using standard methods (Szargut et al., 1988; Ayres et al., 1998).

Finished materials include coal consumed by industry other than electric utilities, gas consumed by households or industry other than utilities, gasoline, heating oil, and residual oil (not consumed by utilities), plus electricity from all sources. Finished non-fuel materials with significant exergy content include plastics, petrochemicals, asphalt, metals, and non-fuel wood. Obviously, large quantities of finished fuels are consumed by industry, for the manufacture of goods, and additional quantities are consumed in transporting those goods to final consumers (i.e. households).

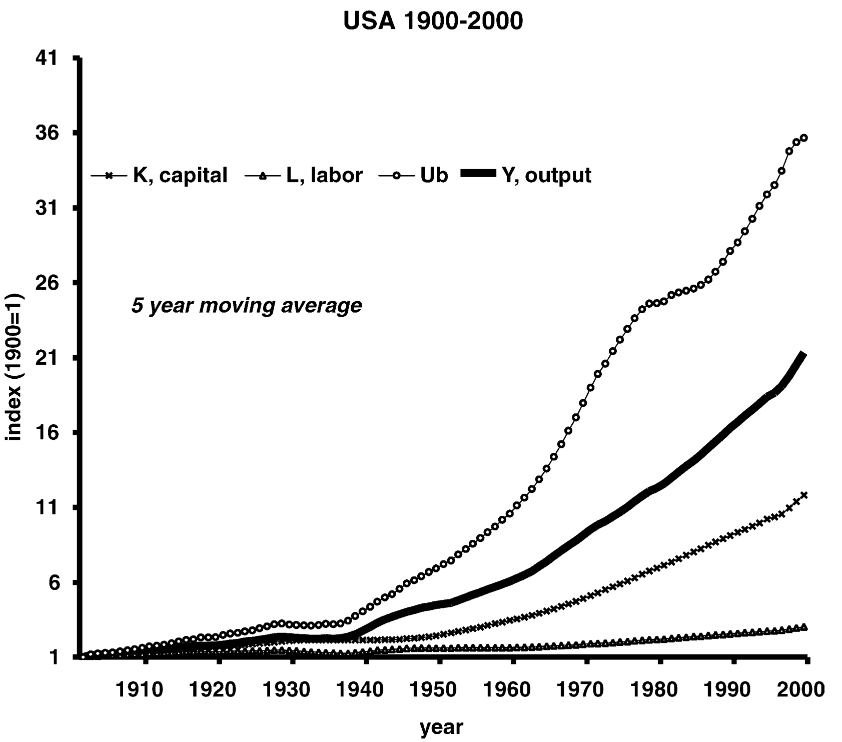

Fig. A.1. Output and factors of production USA 1900–2000.

There are no precise statistics on fuels and materials consumed by ‘final’ users *vis a vis* that which is consumed by intermediates. We do have a breakdown of energy usage since 1955, which distinguishes household use from industrial and commercial use. But transportation use is not subdivided in this way, either by the Department of Energy or the Department of Transportation. The best supplementary source for transportation energy use is Oak Ridge National Laboratory (ORNL). We have rather arbitrarily assigned all gasoline use to households and all diesel fuel use to commercial establishments. This undoubtedly overestimates household use, especially during the early decades of the century before small diesel engines became competitive. There is a further ambiguity, arising from the fact that as much as 40% of all automobile travel is for the purpose of travel to work. It could be argued that this fraction properly belongs to the ‘commercial’ category rather than the ‘private’ category, although we have not done so. Simply, we have calculated the household fraction of all fuels and assumed that the same percentage applies to the exergy content of all final goods.

The major time series for K, L and E, expressed in index form normalized to 1900, are shown in Fig. A.1. Conversion factors are given in Tables A.1 and A.2. Tables A.3–A.6 give the original data sources.

Table A.1
Conversion factors for petroleum products

Substance	Barrels per day to metric tons per year (bd/MT) (IEA Energy Balances, p. vii)	Barrels to million Btu (MBtu/bbl) (EIA Annual Energy Report, Appendix A: Table 1)
Crude oil	50	5.8
Asphalt/road oil	60.241	6.636
Distillate fuel oil	52.356	5.825
Jet fuel/kerosene	46.948	5.67
LPG	31.348	4.13
Motor gasoline	36.496	5.253
Residual fuel oil	55.866	6.287
Lubricants	52.083	6.065
Aviation gasoline	42.918	5.048
Other products	48.077	5.248

Table A.2
Typical chemical exergy content of some fuels

Fuel	Exergy coefficient	Net heating value (kJ/kg)	Chemical exergy (kJ/kg)
Coal	1.088	21680	23587.84
Coke	1.06	28300	29998
Fuel oil	1.073	39500	42383.5
Natural gas	1.04	44000	45760
Diesel fuel	1.07	39500	42265
Fuelwood	1.15	15320	17641

Data source: Expanded from SZAR.

Table A.3
Sources for coal data

Material	Title	Period	Source	Mass (1 short ton = 0.9071847 metric tons)		Heat content (1 Btn = 1055.056 J)	
				Reference	Series name and/or formula	Reference	Formula
Coal (exergy = heat × 1.088)	Raw coal production	1949–1998	Annual Energy Review	Table 7.1, column 1	Production	Table 7.1, column 1; Table A5, column 1	(7.1.1) × (*A*5.1), production
		1850–1948	Historical Statistics, Vol. 1	M93 + M123	Sum “production” = bituminous coal + Pennsylvania anthracite	M77 + M78	Same definition as for mass
	Raw coal apparent consumption	1949–1998	Annual Energy Review	Table 7.1, column 6	“Coal consumption” = production + imports − exports − stock change − losses and unaccounted for	Table 7.1, column 6; Table A5, column 1	(7.1.6) × (*A*5.1), production
		1880–1948	Historical Statistics, Vol. 1	M84, M85 interpolated before 1900	Bituminous consumption in btus/25.4 + anthracite consumption in btus/26.2	M84 + M85 interpolated before 1900	Sum “consumption in Btu”: bituminous coal + Pennsylvania anthracite
		1850–1879	Historical Statistics, Vol. 1	M93 + M123	Consumption assumed = production	M77 + M78	Consumption assumed equal to production
	Coal, apparent consumption as finished fuel	1949–1998	Annual Energy Review	Table 7.1, column 6; Table 7.3, columns 2 and 8; Table 7.7, column 5	Finished fuel = apparent consumption (7.1.6) − coal used in coke plants (7.3.2) − coal used in power plants (7.3.8) + coke consumption (7.7.5)	Table 7.1, column 6; Table 7.3, columns 2 and 8; Table 7.7, column 5; Table A5, columns 1, 3, 7 and 10	Same definition as for mass (7.1.6) × (*A*5.1) − (7.3.2) × (*A*5.3) − (7.3.8) × (*A*5.7) + (7.7.5) × (*A*5.10)
		1916–1948	Historical Statistics, Vol. 1	M85, M84, M116, M114, M122	Finished fuel = apparent consumption (M84/25.4 + M85/26.2) − coal used in coke plants (M116) − coal used in power plants (M114) + coke production (M122 = consumption)	M85, M84, M116, M114, M122	M84 + M85 − (26.8 × M116) − (25 × M114) + (24.8 × M122)
		1890–1915	Historical Statistics, Vol. 1	M85, M84, M114 extrapolated to zero in 1890, M122	Finished fuel = apparent consumption (M84/25.4 + M85/26.2) − coal used in coke plants (l.51 × M122) − coal used in power plants (M114) + coke production	M85, M84, M114 extrapolated to zero in 1890, M122	M84 + M85 − (1.51 × 26.8 × M122) − (25 × M114) + (24.8 × M122)
		1872–1889	Historical Statistics, Vol. 1	M85, M84 interpolated, M122	Finished fuel = apparent consumption (M84/25.4 + M85/26.2) − coal used in coke plants (1.51 × M122) + coke production (M122 = consumption)	M85, M84 interpolated, M122	M84 + M85 − (1.51 × 26.8 × M122) + (24.8 × M122)
		1850–1871	Historical Statistics, Vol. 1	M93 + M123	Finished fuel assumed = production	M77 + M78	Finished = production

Note: Multipliers (26.2, 25.4 and 1.51) derived by exponential fits on years where both series were available.

Table A.4
Sources for petroleum data

Material	Title	Period	Source	Metric tons: M (product) = F (product) × B (product); $F(P)$ = factor (lbs/gal) from Table X for product; $B(P)$ = value in bbl/day × 365 × 42 (gals/bbl)/2204 (lbs/ton)		Heat content (1 Btu = 1055.056 J)	
				Reference	Series name and/or formula	Reference	Formula
Petroleum (exergy = heat × 1.088)	Crude oil production	1949–1998	Annual Energy Review	Table 5.2, column 8	M (crude oil production)	Table 1.2, column 3	Production
		1859–1948	Schurr and Netschert Statistical Appendices	Table A1: I, column 4	M (crude oil production)	Table A1: II, column 4	Production
		1850–1858			Zero		
	Crude oil apparent consumption	1949–1998	Annual Energy Review	Table 5.2, column 8; Table 5.1, columns 5 and 10	M (crude oil production + crude oil imports − crude oil losses) with stock changes + net exports for crude oil per se assumed zero	Table 5.2, column 8; Table 5.1, column 5 and 10; Table A2, columns 1 and 2	M' (crude oil production + crude oil imports − crude oil losses) with stock changes + net exports for crude oil per se assumed zero
		1859–1948	Schurr and Netschert; Statistical Appendices	Table A1: VI, column 4	M (crude oil apparent consumption)	Table A1: VII, column 4	Apparent crude oil consumption
		1850–1858			Zero		
	Petroleum products consumption as finished fuel	1949–1998	Annual Energy Review (EIA)	Table 5.12a, columns 1–5, 7–14; Table 5, 12b, columns 1 and 7	Finished fuel = M (asphalt/road) + M (distillate) + M (jet) + M (LPG total) + M (gasoline) + M (residual) + M (other) for residential/commercial, industrial and transport	Table 2.1, columns 3, 9 and 13	Finished fuel = consumption by residential, commercial, industrial and transport
		1920–1948	Schurr and Netschert Statistical Appendices Historical Statistics, Vol. II	Table A1: VI, column 4; Table 8.8, column 5 (EIA); Table II: S45 (HIST)	Finished fuel = apparent consumption (A1VI.4) − energy sector use (8.85 extrapolated to zero in 1876 using rates from II.S45)	Table A1: VII, column 4; Table 8.8, column 6 (EIA); Table II: S45 (HIST)	Finished fuel = apparent consumption (A1VI.4) − energy sector use (8.85 extrapolated to zero in 1876 using rates from II.S45)
		1850–1858			Zero		

Note on finished fuel calculation: Comparison of values in Annual Energy Review from Table 5.12b (energy sector use) and Table 88 (electric utility use) in common units produce similar numbers for 1949–1998. This suggests that internal use by the petroleum industry of petroleum products has been excluded from apparent consumption. Hence, it has not been subtracted twice.

Table A.5
Sources for natural gas data

Material	Title	Period	Source	Mass (ft^3 = metric tons × 50875.05)		Heat content (1 Btu = 1055.056 J)	
				Reference	Series name and/or formula	Reference	Formula
Natural gas (base units = million ft^3; exergy = heat × 1.04)	Natural gas production includes natural gas liquids	1936–1998	Historical Natural Gas Annual	Table 1, column 1	Gross withdrawals	Table 1, column 1, EIA, A4, column 1	Gross withdrawals (*T*7.1) × dry production factor (*A*4.1)
		1930–1935	Historical Natural; Gas Annual	Table 1, column 5	1.25 × marketed production (1.25 × *T*1.5)	Table 1, column 5, EIA.A4, column 1	1.25 × marketed production × dry production factor (*A*4.1)
		1882–1929	Schurr and Netschert; Statistical Appendix I	Table 1, column 5	1.25 × marketed production (1.25 × *T*I.5)	Constant 1.035 from EIA.A4	1.035 × 1.25 × marketed production
		1850–1881			Zero		
	Natural gas apparent consumption includes natural gas liquids	1930–1998	Historical Natural Gas Annual	Table 2, column 8; Table 1, column 6	Consumption (*T*2.8) + NGL (*T*1.6)	Table 2, column 8; Table 1, column 6; Table A4, columns 1 and 2	Dry consumption (*T*2.8 × *A*4.1) + NGL (*T*1.6 × *A*4.2)
		1882–1930	Schurr and Netschert; Statistical Appendix I	Table VI, columns 5 and 6	Consumption (natural gas + NGL) interpolated 1882–1890	Table VII, columns 5 and 6; Statistical Appendix I	Consumption (natural gas + NGL) interpolated 1882–1890
		1850–1881			Zero		
	Natural gas consumption as finished fuel (excludes NGL)	1930–1998	Historical Natural Gas Annual	Table 3, column 8; Table 3, column 7	Finished fuel = total delivered to consumers (*T*3.8) − electric utility use (*T*3.7) (total deliveries excludes pipeline and plant use). Same as sum (residential, commercial, industrial and transport (*T*3.1 + *T*3.4 + *T*3.5 + *T*3.6)	Table 3, column 8; Table 3, column 7; EIA.A4, columns 3 and 4	Delivered to consumers (*T*3.8 × *A*4.4) − electric utility use (*T*3.7 × *A*4.3)
		1890–1929	Schurr and Netschert Statistical Appendix I Historical Natural Gas Annual	Table 3, column 7 and 8, extrapolated to zero in 1882 using rates from S&N Table VI, columns 5 and 6	Finished fuel = delivered to consumers (*T*3.8 via VI.6) − electric utility use (*T*3.7 via VI.7)	Table 3, columns 7 and 8, extrapolated constant factor 1.035	Finished fuel = 1.035 × (delivered to consumers (*T*3.8 via VI.6) − electric utility use (*T*3.7 via VI.7))
		1850–1881			Zero		

Note: The multiplier 1.25 (marketed for gross) derived from fit on years where both series were available. The constant 1.035 is inferred from all values prior to 1940 in Table A4 of the Natural Gas Annual.

Table A.6
Sources for fuelwood and biomass data

Material	Title	Mass (million cubic feet roundwood equivalent × (0.017–0.022) = multiplier time dependent (MMT)			
		Period	Source	Reference	Formula
Fuel wood (exergy = heat × 1.152)	Fuelwood production = consumption = consumption as finished fuel	1997–1998	Annual Energy Review	Table 10.3, row 1	Wood energy (Btu) × 1535
		1965–1996	Statistical Abstract	Table 1152, last row	Fuelwood consumption (mcfre) × multiplier
		1958–1964	Interpolation		
		1900–1957	Potter and Christy	Table FO-13, column B	New supply fuelwood × multiplier
		1850–1899	Schurr and Netschert	Table 7, column 1	5-year interpolations × multiplier
		Heat content (1 Btu = 1055.056 J)			
		Period	Source	Reference	Formula
		1981–1998	Annual Energy Review	Table 10.3, row 1	Wood energy
		1970–1980		Table 10.3, row 1 and Table 1.2, column 10	Wood energy and energy from biomass, adjusted and interpolated
		1949–1969		Table 1.2, column 10	Energy from biomass (= fuelwood only)
		1850–1949	Historical Statistics, Vol. 1	M92, interpolated	Fuel wood consumption

References:

Sources have often been abbreviated in the tables—these abbreviations are shown in italic capitals at the end of each relevant citation. Brackets indicate the main textual citation.

ANNERO: *Annual Energy Review* (USDOEEIA, annual).
BEA: *Long Term Economic Growth 1860–1970* (USDOCBEA, 1973).
BUSTAT: *Business Statistics* (USDOC, 1992).
CEA: *Economic Report of the President* (USCEA, 1996).
EIA = *Annual Energy Review* (USDOEEIA, annual).
HISTAT: *Historical Statistics of the United States: Colonial Times to 1970, 1975* (USCensus, 1975).
HNGA: *Historical Natural Gas Annual* (USDOEEIA, 1999).
IEA: *Energy Balances of OECD Countries*, annual (OECD/IEA, 1999).
MINYB: *Minerals Yearbooks*, annual (USGS, 1999; USBuMines annual).
P&C: Potter and Christy (Potter and Christy, 1968).
S&N: Schurr and Netschert (Schurr and Netschert, 1960).
SCB: *Survey of Current Business*, monthly (USDOCBEA).
STATAB: *Statistical Abstract of the United States* (STATAB annual).
Szar: Szargut (Szargut et al., 1988).

Appendix B

The LINEX function parameters were obtained by using a quasi-Newton non-linear optimization method, with box-constraints. The objective function was simply the sum of the squared error. The constraints on the possible values of the parameters of the LINEX model were required to ensure that the factor marginal productivities (Eqs. (4a)–(4c)) were non-negative. A statistical measure of the overall fit was provided by the mean square error

$$\mathrm{MSE} = \frac{\sum_{t=1}^{n}(e^2)}{n-k} \tag{B.1}$$

The MSE is a measure of the absolute deviation of the theoretical fit from the empirical curve, where n is the number of samples, k the number of parameters and e the residual from the fitted curve.

To compare the predictive power of raw exergy flows (B and E) *vis a vis* physical work (U_B and U_B) we calculated the correlation coefficients (R^2) between the logarithms of the LINEX function of the variables and the actual GDP. We tested the significance of the correlations using a t-test with Welch modification for unequal variances (Table B.1). The results showed that the relationship of GDP with a LINEX function of B or E was not significant. Substituting U_E and U_B the correlations were significant, and the latter choice was by far the most significant, as indicated by the small t-value.

The coefficient of determination is often reported as another measure of the goodness of fit. To be valid the residuals should be identically and independently distributed. The

Table B.1
Statistical measures of the quality and significance of fitted models

Variable used	t-value	Degrees of freedom	P-value	Correlation coefficient (R^2)
B	5.74	158.4	4.5E−08	0.98
E	3.09	174.3	0.002	0.97
U_E	0.51	194.8	0.604	0.99
U_B	0.19	195.9	0.845	0.99

Durbin–Watson statistic was used to check for the presence of correlated residual error. It is calculated as

$$\mathrm{DW} = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n}(e^2)} \tag{B.2}$$

where e is the residual error, calculated for each year t, for a time period of length n, where k is the number of independent variables. The DW statistic takes values between 0 and 4. The Null Hypothesis was that there was no significant correlation between the residual error values for each year.

The DW statistic, calculated for each estimate using either definition of work input (U_E or U_B) for the entire period (1900–1998), provided evidence of strong residual autocorrelation (DW < 1.61, $k = 3$). The Null Hypothesis was therefore rejected.

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